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### Semester One Examination, 2020

### Question/Answer booklet

# MATHEMATICS

**SOLUTIONS**

**APPLICATIONS**

**UNIT 3**

## Section One:

## Calculator-free

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| WA student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

|  |  |
| --- | --- |
| Number of additional answer booklets used (if applicable): |  |

## Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| Section One: Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

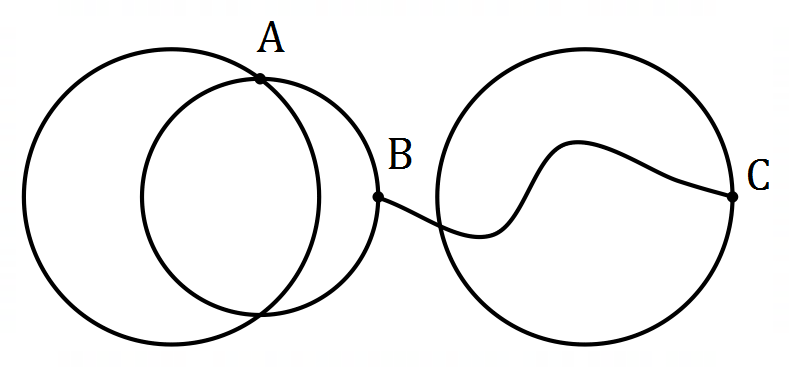
Section One: Calculator-free 35% (52 Marks)

This section has**eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 (6 marks)

Consider the following graph .



(a) Draw in the plane, to clearly show that it is planar. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ 5 edges, 3 labelled vertices  ü correctly drawn in the plane |

(b) State the degree of each vertex and the degree sum for . (2 marks)

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| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct degrees  ü correct degree sum |

(c) Explain how to recognise a bridge in a connected graph and state, with justification, whether contains a bridge. (2 marks)

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| **Solution** |
| When a bridge is removed from a connected graph, the graph becomes disconnected.  Yes, edge is a bridge in . |
| **Specific behaviours** |
| ✓ correct explanation  ü identifies edge that is bridge |

Question 2 (6 marks)

A recursive rule for a sequence is .

(a) Briefly explain which feature of the recursive rule indicates that the sequence is arithmetic.

(1 mark)

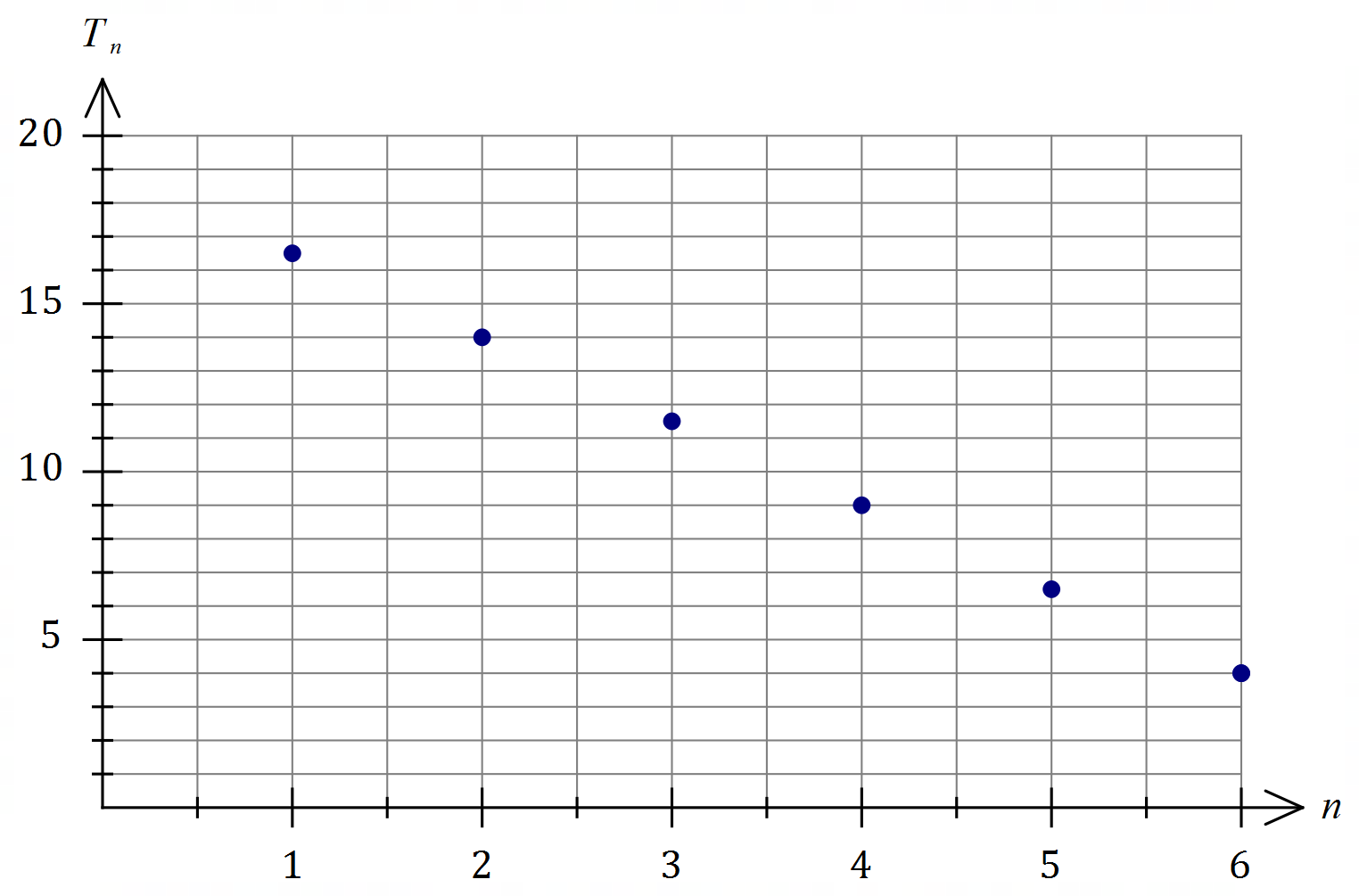
|  |
| --- |
| **Solution** |
| There is a constant difference of between consecutive terms. |
| **Specific behaviours** |
| ✓ indicates the constant difference |

(b) Determine and . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ value of  ü value of |

(c) Graph the first six terms of the sequence on the axes below. (2 marks)

|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ at least correct points  ü all correct, no line |



(d) What feature of the graph indicates that the sequence is arithmetic? (1 mark)

|  |
| --- |
| **Solution** |
| The points are linear / lie in a straight line. |
| **Specific behaviours** |
| ✓ indicates linear nature |

Question 3 (6 marks)

(a) Connected planar graph has vertices and edges.

(i) Use Euler's formula to determine the number of faces in . (2 marks)

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| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct use of Euler's formula  ü correct number of faces |

(ii) Sketch a possible graph . (2 marks)

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| **Solution** |
| (Example only - others exist) |
| **Specific behaviours** |
| ✓ correct number of edges and vertices  ü connected, with correct number of faces |

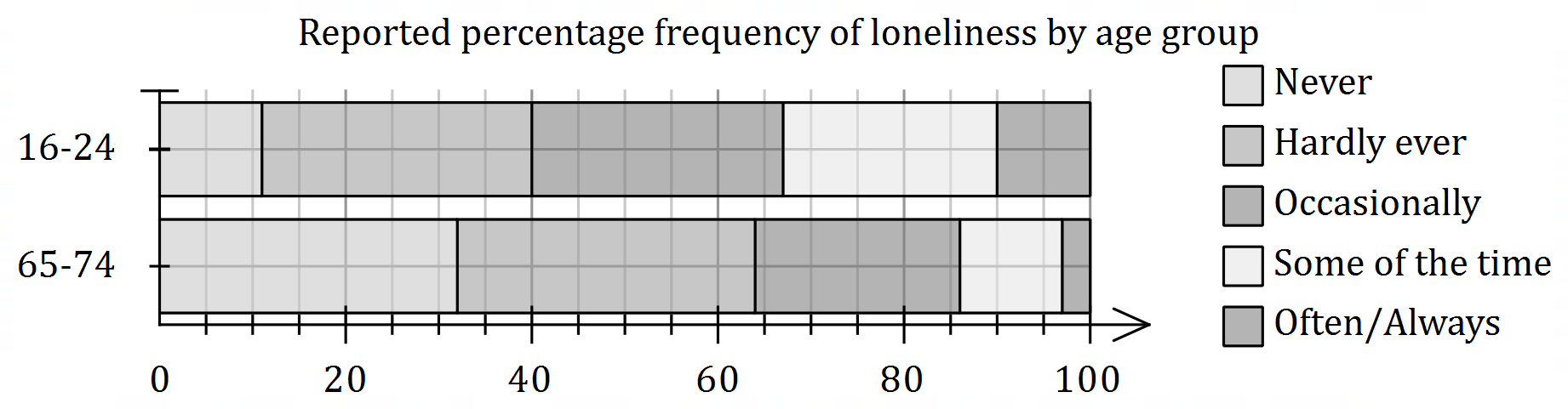
(b) Graph has vertices and is Eulerian. The length of the Eulerian trail is . Sketch a possible graph . (2 marks)

|  |
| --- |
| ***Markers note*** |
| *Penalise mark overall if vertices not clearly identified.* |

|  |
| --- |
| **Solution** |
| (Example only - others exist) |
| **Specific behaviours** |
| ✓ connected with edges and vertices  ü all vertices with even degree |

Question 4 (7 marks)

A recent survey asked people their age and the question "How often do you feel lonely?". The responses for the age groups years and years are shown below. The categories in the key are shown from left to right in the stacked percentage frequency graph.



(a) State the second largest category for the age group. (1 mark)

|  |
| --- |
| **Solution** |
| 'Occasionally'. |
| **Specific behaviours** |
| ✓ correct category |

(b) State the percentage of those

(i) in the age group who responded with 'Some of the time'. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct percentage |

(ii) in the age group who responded with 'Hardly ever' or 'Occasionally'. (1 mark)

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| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct percentage |

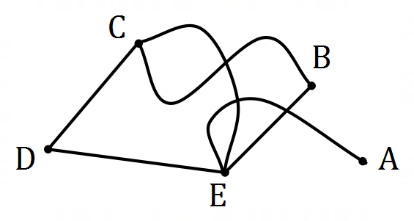
(c) Use the graph to explain why the data suggests an association exists between feeling lonely and age. (2 marks)

|  |
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| **Solution** |
| Only of age group never feel lonely compared to of the age group. The difference in these percentages suggests an association. |
| **Specific behaviours** |
| ✓ example using correct percentages  ü indicates different percentages suggest association |

(d) Briefly discuss non-causal explanations for the observed association. (2 marks)

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| **Solution** |
| The association may be due to coincidence (possibly only a small, biased survey, etc) or due to confounding by another variable (possible that one age group lived in city and the other in rural area, etc). |
| **Specific behaviours** |
| ✓ indicates coincidence  ü indicates confounding |

Question 5 (6 marks)



Graph is shown. and are subgraphs of , so that each subgraph has vertices but one less edge than .

(a) does not satisfy Euler's formula. State which edge must be removed from , and show that does not satisfy Euler's formula. (3 marks)

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| --- |
| **Solution** |
| Remove edge .  Substituting the values into the expression then but Euler's formula states this value must equal . Hence graph does not satisfy Euler's formula. |
| **Specific behaviours** |
| ✓ correct edge to remove  ✓ correct values of  ü substitutes and shows formula not satisfied |

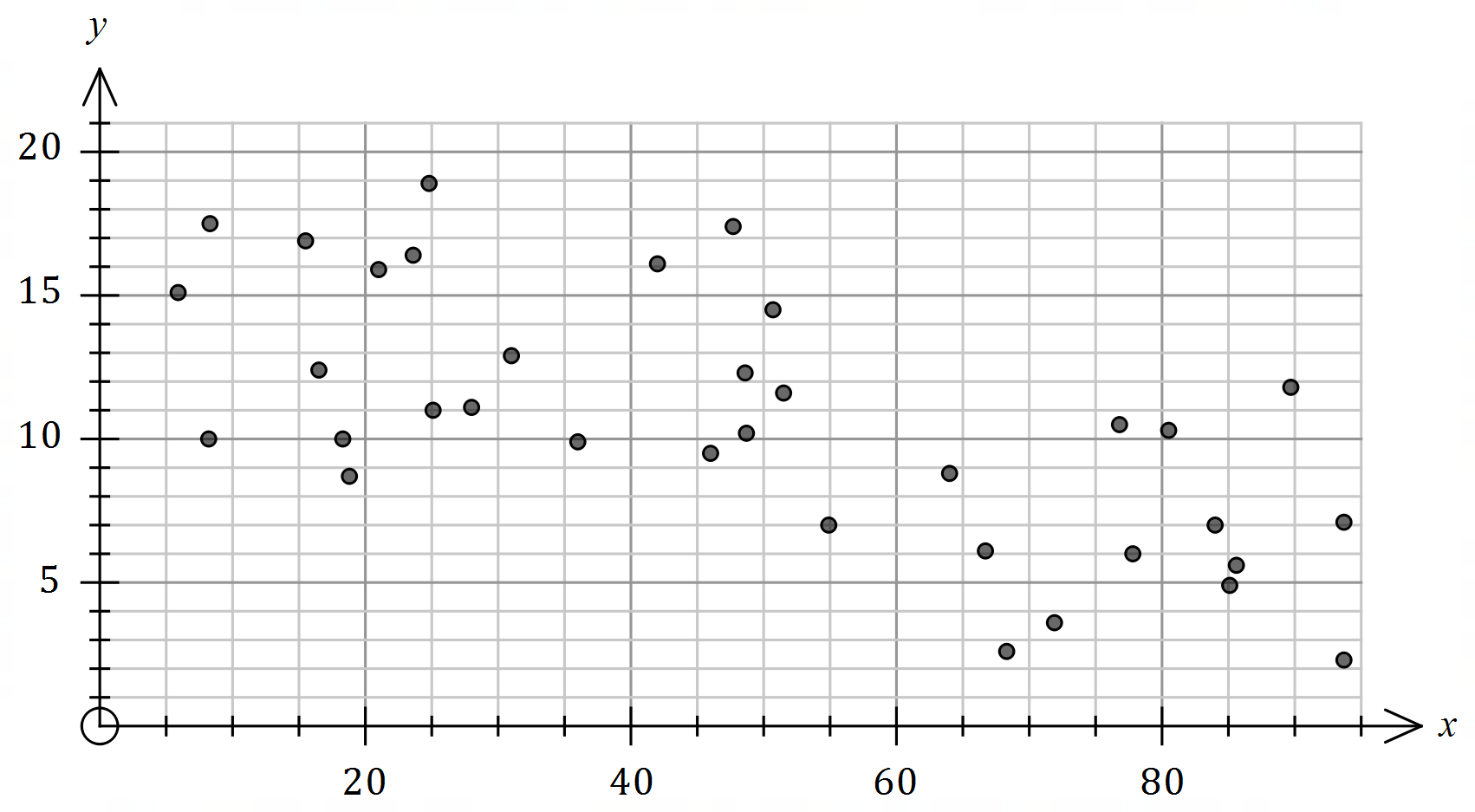
(b) is bipartite. State which edge must be removed from , and draw to clearly show the partite sets. (3 marks)

|  |
| --- |
| **Solution** |
| Remove edge .    *(Bipartite graphs have no odd length cycles)* |
| **Specific behaviours** |
| ✓ correct edge to remove  ü correctly shows partite sets  ü correctly draws edges |

Question 6 (7 marks)

The scatterplot below shows the number of PC's per people on the -axis and the

unemployment rate, as a percentage, on the -axis for a selection of countries in .



(a) Describe the strength and direction of the association between the variables. (2 marks)

|  |
| --- |
| **Solution** |
| The association is of moderate strength and in a negative direction. |
| **Specific behaviours** |
| ✓ strength  ü direction |

(b) The equation of the least-squares line for the data is . Interpret the intercept and the slope of this line. (3 marks)

|  |
| --- |
| **Solution** |
| The intercept means that if a country had no PC's then the unemployment rate is expected to be .  The slope means that for every extra PC per people, the unemployment rate is expected to fall by . |
| **Specific behaviours** |
| ✓ interprets intercept  ü interprets slope  ü includes units (%) in interpretations |

(c) A newspaper article used the graph to claim that increasing the number of PC's per person in a country caused the unemployment rate to fall. Comment on this claim.

(2 marks)

|  |
| --- |
| **Solution** |
| The claim is unlikely to be true - an observed association does not necessarily mean there is a causal relationship between variables. |
| **Specific behaviours** |
| ✓ disputes validity of claim  ü comments on causation |

Question 7 (7 marks)

In a set of pages on a website, there are hyperlinks from page to page , from page to  
page , from page to page , from page to pages and , and from page to page .

(a) Construct digraph to show the above information, where pages are represented by vertices and links by directed edges. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ vertices all have correct degree  ü edges all directed correctly |

(b) Construct an adjacency matrix for . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ at least correct rows  ü all correct |

(c) List, starting at page and in the order visited, vertices in that form a

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct walk |

(i) walk of length . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct cycle |

(ii) cycle of length . (1 mark)

(iii) trail of length . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct trail |

Question 8 (7 marks)

Every day, of the water in a tank is drained for crop irrigation and then the tank is topped up with kL of water. The tank has a maximum capacity of kL.

At the start of Day , before water is drained for the crops, the tank contains kL.

(a) Determine the amount of water in the tank at the start of Day . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ amount drained  ü correct amount |

(b) Determine a recursive rule for the amount of water, , in the tank at the start of Day .

(2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct multiplier  ü correct addition and first term |

(c) Explain why the tank will never empty. (1 mark)

|  |
| --- |
| **Solution** |
| With this type of recursive rule, the amount in the tank will always increase, tending towards a steady state amount. OR  Amount drained is always less than amount added. |
| **Specific behaviours** |
| ✓ indicates amount always increasing |

(d) State, with justification, whether the tank will overflow. (2 marks)

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| --- |
| **Solution** |
| Let be long-term steady state.  Hence tank will overflow, as the capacity of the tank ( kL) is less than the long-term steady state ( kL). |
| **Specific behaviours** |
| ✓ indicates correct long-term steady state  ü states tank will overflow, with justification |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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